17. State and prove uniform limit theorem.

18. Prove that the real line R is connected.

19. Let X be a nonempty compact Hausdorff space. If X has no isolated points then prove that X is uncountable.

20. State and prove Tietze Extension theorem.

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NOVEMBER/DECEMBER 2024

GMA32/DMA32 — TOPOLOGY

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Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define order topology.

2. Define interior of a topological space with example.

3. Define homeomorphism.

4. What is meant by metric topology?

5. Define path connected space.

6. Describe the components of X.

7. Define sequentially compact space.

8. State tube lemma.

9. When do you say the set in dense?

10. Define separable space.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Let $\mathcal{B}$ and $\mathcal{B}'$ be based for the topologies τ and τ' respectively on X. Then prove that the following are equivalent.

(i) τ' is finer than τ

(ii) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is basis element $\mathcal{B}' \in \mathcal{B}'$ such that $x \in \mathcal{B}' \subset B$.

Or

(b) Let A be a subset of the topological space X. Let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$.

12. (a) Let X be a metric space with metric d. Define $\overline{d} : X \times X \to R$ by $\overline{d}(x, y) = \min\{d(x, y), 1\}$ then prove that $\overline{d}$ is a metric that induces the same topology as d.

Or

(b) State and prove pasting lemma.

13. (a) Let A be a connected subspace of X. If $A \subset B \subset \overline{A}$ then prove that B is also connected.

Or

(b) State and prove intermediate value theorem.

14. (a) Prove that the image of a compact space under a continuous map is compact.

Or

(b) State and prove extreme value theorem.

15. (a) Prove that every metrizable space is normal.

Or

(b) Let X be a topological space. Let one point sets in X be closed. Then prove X is regular if and only if given a point x of X and a neighbourhood U of x there is a neighbourhood V of x such that $\overline{V} \subset U$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let Y be a subspace of X. Let A be a subset of Y. Let $\overline{A}$ denote the closure of A in X. Then prove that the closure of A in $Y = \overline{A} \cap Y$.

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